DETECTION THEORY APPLIED TO THE ABSOLUTE SENSITIVITY OF SENSORY SYSTEMS

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ABSTRACT Mainly the skin senses touch and warmth have been investigated. It is shown that the decision model describes the experimental data better than the threshold model. The experiments lead to the assumption that an internal noise exists, which is a neural activity being undistinguishable from the neural activity caused by small stimuli and which adds to the neural activity caused by the stimulus. The probability distribution of this internal noise can be considered to be gaussian. The relation between stimulus and neural activity is alinear for the touch sense. The question of whether noise of a multiplicative nature must be assumed is discussed.

INTRODUCTION

The modern theory of signal detectability (1, 2) has been successfully applied to the sensory detection of signals (3, 4). This theory describes certain aspects of sensory perception much better than the conventional threshold theory (5-7).

In the simple version of the latter the threshold is a fixed level of signal strength or of the corresponding neural activity, below which no detection of the signal can occur, while above this level the signal is detected with certainty. In order to explain the shape of the frequency of detection curve plotted as a function of signal strength it is assumed that either the signal varies statistically (*e.g.* numbers of quanta in a light flash for the visual perception), or that the threshold level varies, or both. Possibly existing internal noise of an additive nature which is a neural activity in the sensory channel unrelated to the presence of a signal, is not considered in threshold theory.

In the theory of signal detectability, which is based on statistical decision theory, it is assumed that additive noise exists and that every level of noise or signal-plus-noise however small, can be detected. In the so called yes-no experiments the observer makes a decision about presence or absence of a signal by adopting a decision level or criterion which is adjustable. By adjusting this level the observer can strive after an optimum detection, which is a well defined concept in decision theory.

This theory has been mostly used for description of the observation of signals in the presence of purposely added noise. Especially the detection of sound, sine wave, or noise, in band-limited gaussian noise has been investigated in the light of detection theory (6, 8-11).

One of the main advantages of the decision model as compared with the threshold model appears to be that in the former a parameter for the signal detectability can be defined which can be experimentally determined and which is independent of the psychophysical procedure used. The shift of the decision level of an observer striving after optimal detection caused by a change in the detection situation is predicted by the detection theory. In contrast, the variation of the threshold value, with the psychophysical procedure is unaccountable in the threshold model.

The occurrence of a positive response, when no signal is presented during an observation interval, a so called false positive, is expected on the grounds of the detection model. The threshold model can interpret these false positives only as mere guesses, *i.e.* a response, which is not based on any information obtained from the presence or absence of a signal.

The application of detection theory to the experimental determination of the absolute sensitivity of a sense organ requires the assumption that internal noise of an additive nature exists. This internal noise is a neural activity in the sensory channel not due to the stimulus. It adds to the neural activity caused by the stimulus. This assumption is strongly supported by electrophysiological data. But the properties of this internal noise are almost entirely unknown.

As the decision model has distinct advantages over the threshold model it seems worthwhile to try to obtain more knowledge about the internal noise. The noise can be expressed in terms of equivalent input signals and as the input signal is the only quantity in psychophysical experiments which can be physically measured and which can be compared with the internal noise, it will be often necessary to use this measure. But as will be shown in this paper it is useful to think of internal noise as a neural activity and it is possible to derive valuable information about its distribution.

The model used in this paper is undoubtedly too simple for an ultimate description of the detectability of small signals by human observers. But at this stage of knowledge this is the only way to attack this problem. It is assumed that a physical signal gives rise to a neural activity in the sensory channel and that also internal noise exists. Detection takes place on the basis of the total magnitude of the neural activity, which is characterized by a single quantity during the observation interval. The relation between the magnitude of the signal and the thereby caused neural activity is monotonic but needs not to be linear. Variation of this relation which is a kind of multiplicative noise is only considered when the experimental results contradict the detection model.

METHODS

The psychophysical experiments mentioned below are of three types. In the first type one of five stimuli of different amplitudes is presented to the observer in a given time interval. The stimuli are given in random sequence. One of the stimuli has zero amplitude. The response of the observer can be either yes or no indicating whether or not after his opinion a stimulus has been given. No costs or values are announced to the subject for a correct response, a false alarm or a missed signal. The fact that the stimuli are given at random results in the subject adopting a criterion, which on the average is the same for all stimuli. The aim of these experiments was to investigate the probability distribution of the noise, including the fluctuation of the criterion, by analysis based upon the decision model.

In the second type of experiments the observer has to choose one out of four possibilities. In an observation interval one of four stimuli with different amplitudes is presented to the observer, and the observer responds by stating which of the four stimuli he thinks is given. The observer is trained to the set of stimuli before the proper measurements start. The magnitudes of the stimuli are chosen in such a way that a probability of detection between 0.05 and 0.95 for each stimulus is obtained. As will be explained below this kind of experiment allows conclusions about the linearity or non-linearity of the relation between stimulus strength and neural activity.

Finally, the third type of experiments is of the forced choice type. In one of four observation intervals a stimulus of known magnitude is presented and the observer is asked to indicate in which interval he thinks the stimulus has been given. The observer must also make a second choice different from the first one. With the results of these measurements the important question can be answered whether the false positive responses (saying yes when no stimulus is offered) are observations of noise undistinguishable from the signal or mere guessing. This is the crucial question for a decision between the threshold model and the decision model.

The occurrence of a stimulus interval is signalled to the observer by sound or a light flash. It is secured that stimuli of different amplitudes differ only in amplitude, everything else being the same. Constancy of time course of the stimuli is important because of the dynamic properties of the sensitivity of the skin senses (12-14). All observers had thorough training before the final experiments were carried out.

The main experiments were carried out on the skin senses of touch and warmth. The stimuli were deformation, electrical current, and increase of temperature. The site of stimulation was the inner side of the forearm. A few experiments were performed on the auditory and visual sense organs. For the experiments described in this paper only the relative values of the signal strengths need be known. The relevant measure of the signal strength is considered to be the deformation for the stimulation of the touch, the electrical current strength for the electrical stimulation, the voltage across the earphone for the auditory experiments and the light energy for the visual experiments. The adequate stimulus of the warmth sense is best expressed in terms of temperature increase of the skin (15).

The touch stimuli were linearly increasing deformations with duration of 0.16 second or pulse-shaped with duration of 70 msec. The electrical stimuli were pulse-shaped with a duration of 2 msec. The warmth stimuli were given by irradiation with infrared of constant intensity resulting in a linearly increasing temperature of the skin surface during the exposure time of 0.38 second and 1.2 seconds, respectively. To prevent too much increase of temperature of the skin the time between two stimuli was more than

15 seconds. The experiments on hearing were done in an anechoic room. Sine wave stimuli of 1000 CPS with a duration of 0.2 second were applied monaurally.

The observer determines the starting time of a trial by pressing a button. For a part of the experiments an electronic spinning disc was used producing a random choice out of four possibilities.

THEORY AND RESULTS OF THE EXPERIMENTS

Distribution of Internal Noise. In Fig. 1 the probability density of the neural activity x is plotted when, respectively, noise alone and signal-plus-noise are present. The magnitude of the neural activity determines whether a positive or negative response is given. If the neural activity during an observation interval is

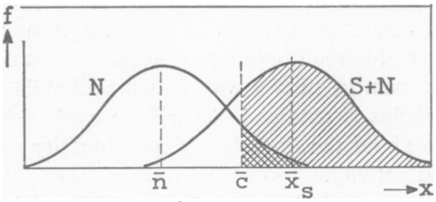

FIGURE 1

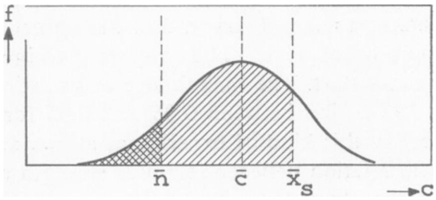

FIGURE 2

FIGURE 1 Probability density of the neural activity x, when, respectively, noise alone and signal-plus-noise are present. $\bar{c}$ is the criterion.

FIGURE 2 Model equivalent with Fig. 1.

larger than a criterion c a positive response is given; if it is smaller, a negative response occurs. One of the underlying assumptions of the decision model used is that the signal itself is not noisy. The two distributions differ only in the average value.

The criterion c may not be constant. But the fluctuations of the criterion can be included in the fluctuation of the noise. It can be shown that a gaussian noise with mean $\bar{n}$ and variance σ_n^2 and a criterion with also a gaussian distribution with mean $\bar{c}$ and variance σ_c^2 is mathematically equivalent to a gaussian noise with mean $\bar{n}$ and variance $\sigma^2 = \sigma_n^2 + \sigma_c^2$ and a constant criterion $\bar{c}$.

Another equivalent model is a constant noise with value $\bar{n}$ and a fluctuating criterion with mean $\bar{c}$ and variance $\sigma^2 = \sigma_n^2 + \sigma_c^2$. The variable quantity in this second model is the criterion c (Fig. 2). This c is equal to $- x + \bar{c} + \bar{x}$ in the first model, where $\bar{x} = \bar{n}$ or $\bar{x}_s$. The equivalency of both models can easily be seen by comparing the Figs. 1 and 2 and can be proved by simple mathematics. We will now use the second model. This way of description is somewhat easier to handle when various signals with different strengths are randomly applied. The probability density of the criterion includes the variability due to the fluctuations of the noise.

If we assume that signal strength s and x are linearly related the probability density function will have the same shape when plotted *versus* s as plotted *versus* x or c. The experiments of the first type are used for the measurement of this distribution function. Five signals including a signal of zero amplitude are presented at random to the observer. The observer responds by yes or no indicating whether he thinks a signal has been given or not. The five signals are equally probable.

These experiments have been carried out on the senses of warmth, touch and hearing and with electrical stimulation of the skin. The results are partly shown in Figs. 3, 4, and 5. The probability is plotted on the ordinate which has a gaussian probability scale.

It will be seen that in first approximation the experimental points of the senses of warmth and touch lie on a straight line which means that the distribution of the probability density is gaussian. The standard deviation σ of this distribution is the best measure of the sensitivity of the sensory system. In our experiments the value of σ of the warmth sense was about 0.04°C, the σ of the touch sense about 60μ. But it must be emphasized that these figures are dependent on many parameters as size and shape of the stimulus, place of stimulation, and also on the time course of the stimulus (12-14).

Taking into account all the measurements on various subjects it appears that for the warmth sense no significant deviation from the gaussian distribution is found. However, the results of the experiments on the senses of hearing and touch and with electrical stimulation show a deviation, which is very marked in the electrical stimulation experiments. The probability of response on the smallest signal, which is in this case a signal of zero amplitude, is too high. This deviation can be due to the following causes. First the transfer function $g(s)$ which transforms the signal s into the neural activity x, $x = g(s)$, may be fluctuating. This would be a kind of multiplicative noise, while the noise mentioned so far is of an additive nature. The influence of this fluctuation would, however, very likely increase with signal strength. This results in an increase of the standard deviation of the probability density function with signal strength. The experimentally found deviation of the normal distribution, however, is entirely otherwise. Therefore this influence cannot explain the experimental results. Secondly the relation between signal strength and neural activity x may be non-linear or thirdly this relation being linear the distribution plotted *versus* x is skew (Fig. 6).

Skewness of Distribution or Alinearity. To design an experiment which offers the opportunity to distinguish between a skew distribution and non-linearity of the relation between signal strength and neural activity the following consideration is relevant. In experiments of the first type the distribution of the criterion is determined by measuring the probability of correct responses to signals of various strengths. If one could shift the distribution of the criterion in respect to the strength of the signals and again could determine this distribution by the same measurement a distinction between skewness and alinearity could be made. This is done in the following way.

In an observation interval one of four signals S_1, S_2, S_3, S_4 with different strengths,

$s_1 < s_2 < s_3 < s_4$ is presented to the subject. The subject must respond by saying which signal of the four he thinks is given. The subject needs for this response three criteria c_1, c_2, and c_3. If one of the four signals, say S_i, is presented, a neural activity x_i is caused. The subject responds by saying "four" (R_4 occurs) if $x_i > c_3$; and he responds by saying "three" (R_3 occurs) if $c_2 < x_i < c_3$. So R_2 occurs if $c_1 < x_i < c_2$ and R_1 if $x_i < c_1$.

The responses R_4 on the various signal strengths provide the distribution of c_3 as a function of signal strength. The responses $R_4 + R_3$ which occur if $x_i < c_2$ give in the same way the distribution of c_2 and $R_4 + R_3 + R_2$ yield the distribution of c_1. The obvious assumption is made that always $c_1 < c_2 < c_3$.

In Figs. 7 and 8 the consequences of the two possibilities, skewness or non-linearity, for the relationship between the so measured distributions of c_1, c_2, and c_3 are shown. If the distribution is skew the three curves plotted on probability paper are only shifted in horizontal direction. This is obvious as the three distribution curves, c_1, c_2 and c_3 in the upper part of Fig. 7 are equal apart from a horizontal shift. If non-linearity, however, exists the curves are in first approximation mutually shifted in vertical direction. This is illustrated in Fig. 8. As the distribution curves c_1, c_2, and c_3 are now assumed to be symmetric gaussian curves the cumulative distribution curves plotted on probability paper as a function of c are straight lines. These lines are drawn in the bottom part of Fig. 8 partly solid, partly broken. However, plotted as a function of signal strength the solid curved lines are obtained if

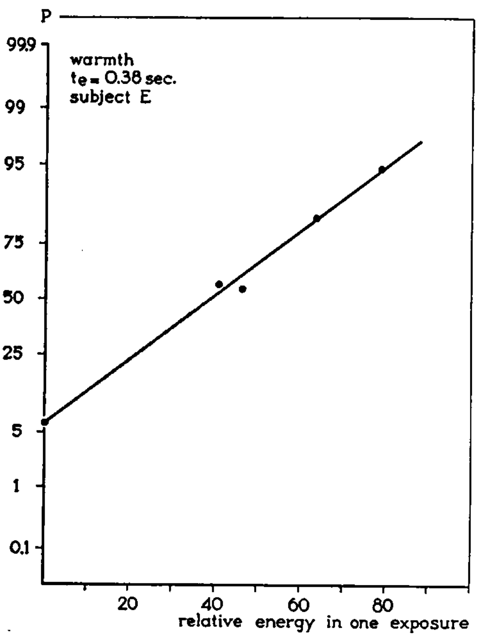

FIGURE 3

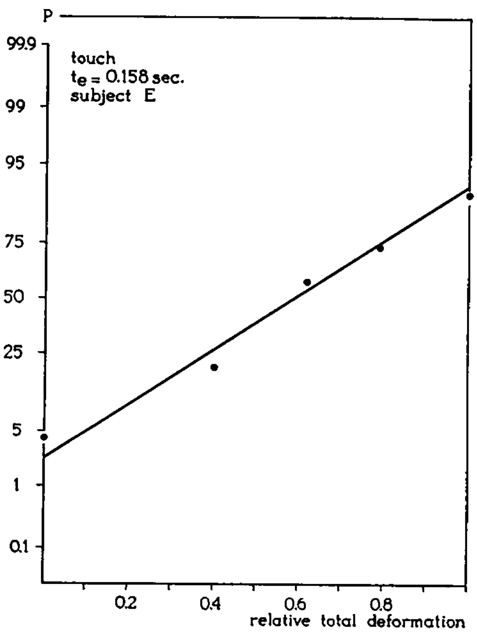

FIGURE 4

non-linearity between neural activity and signal strength exists as assumed in Fig. 8. In Figs. 6, 7, and 8 it is assumed for the sake of simplicity of the explanation that the signal strengths are equally spaced along the strength axis.

These experiments have been performed with the sense of touch and with electrical stimulation. Some of the results are shown in Figs. 9 and 10. Of the two possibilities considered the hypothesis of non-linearity explains the experimental results, while skewness of the distribution is clearly excluded.

Second Forced Choice. If noise in the neural channel exists which is undistinguishable from the activity caused by a signal, the subject will make also false positive responses. However, the existence of false positives does not prove the existence of this kind of noise. It could be that these false positives were mere guesses. Guesses are positive responses which are made without using any information from the fact that in the observation interval a signal is presented or not.

If the threshold model were valid false positives would only be guesses. The experiments described above and other similar experiments indicate, however, that false positives should be explained on the basis of the decision model and should not be considered as guesses.

Additional evidence on this point can be obtained by the following experiment, which has been carried out by Swets with visual signals (5, 7). In a four alternative forced choice experiment the subject is asked not only to state in which of the four intervals the signal is given but also to give a second choice. Assuming that the

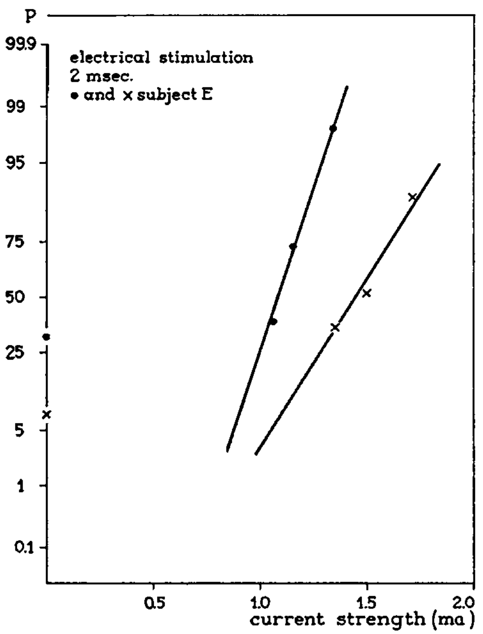

FIGURE 5

FIGURES 3, 4, AND 5 Relative frequency of positive response *versus* signal strength plotted on probability paper.

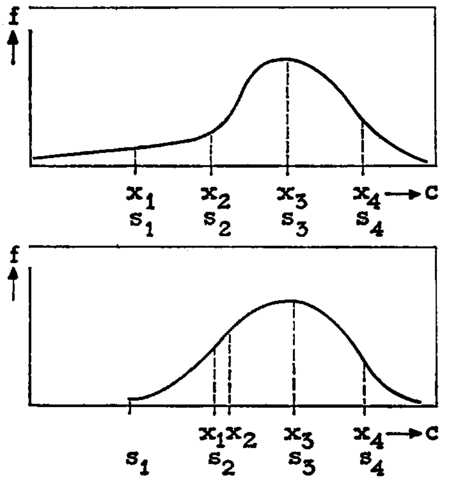

FIGURE 6 Above: Probability density curve is skew. Criterion c and signal strength s linearly related. Below: Symmetrical probability density curve, relation between c and s non-linear.

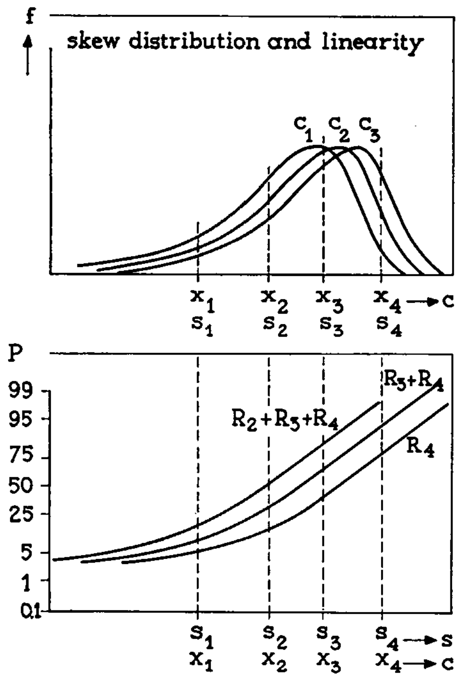

FIGURE 7

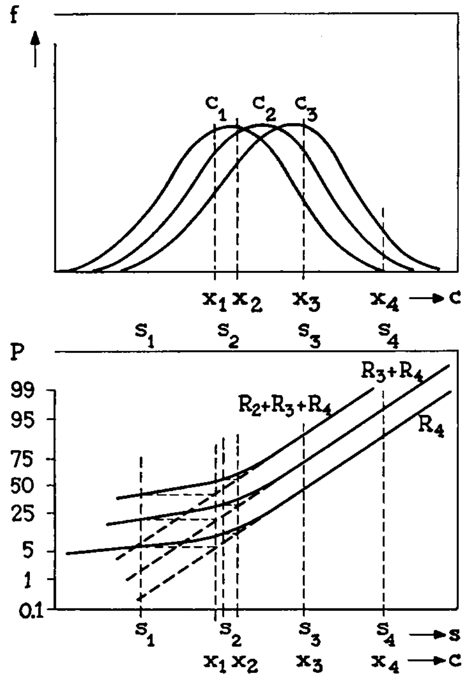

FIGURE 8

FIGURE 7 Four observation categories. Skewness causes the probability *versus* signal strength s curves to shift in horizontal direction.

FIGURE 8 Four observation categories. Symmetrical probability density curve and non-linear relation between criterion c and signal strength s causes shift in vertical direction.

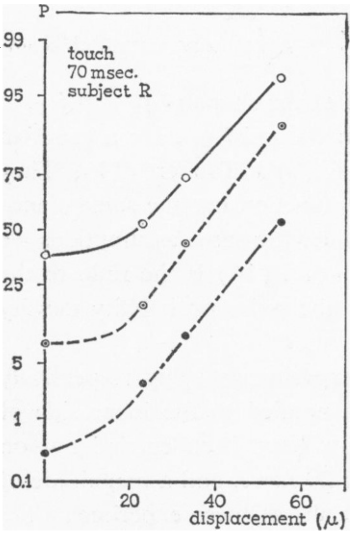

FIGURE 9

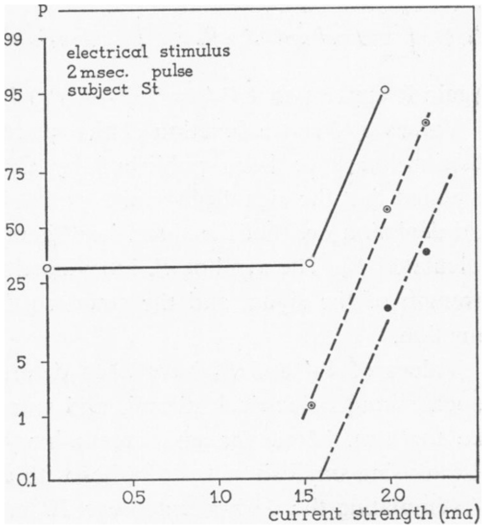

FIGURE 10

FIGURE 9 Experiment with four categories on touch. Curves are shifted in vertical direction.

FIGURE 10 The same as Fig. 9, now with electrical stimulation.

decision model holds a correct first choice response will occur if the neural activity due to signal-plus-noise is larger than the neural noise in the three other intervals. A correct second choice response will occur if the neural activity due to signal-plus-noise in one interval is smaller than the neural noise in another interval and larger than the neural noise in the remaining two intervals.

If the threshold model is valid the second choice response can only be a guess. When the first choice is incorrect the probability of a correct second choice will be then 0.33, independent of signal strength.

The probability of the correct first choice and second choice in the four alternative experiments will be called P_4 and P_3' respectively. The probability of the correct first choice in an experiment, which differs only from the former one in using three observation intervals, in one of which the signal is given, will be called P_3. It can be shown that very generally the relation holds $P_3' = 3(P_3 - P_4)$.

As mentioned above the threshold model predicts that $P_3' = 1/3(1 - P_4)$. If the probability of a correct response in a yes-no experiment with the same signal is called P then $P_3 = 2/3\ P + 1/3$ and $P_4 = 3/4\ P + 1/4$. It follows from these three relations that $P_3' = 3(P_3 - P_4)$.

Applying detection theory and calling the probability density of the noise as a function of neural activity f_N the one of neural activity due to signal-plus-noise f_{SN} (Fig. 1) and the corresponding cumulative distributions P_N and P_{SN} respectively, one obtains

$$P_4 = \int_{-\infty}^{+\infty} f_{SN} P_N^3 \, dx, \quad P_3 = \int_{-\infty}^{+\infty} f_{SN} P_N^2 \, dx \quad \text{and} \quad P_3' = 3 \int_{-\infty}^{+\infty} f_{SN}(1 - P_N) P_N^2 \, dx.$$

Again it appears that $P_3' = 3(P_3 - P_4)$ independent of the shape of f_N and f_{SN}.

Values of P_4 as a function of the quantity d' when P_N and P_{SN} have a gaussian distribution have been published by Green, Birdsall, and Tanner (11). They assumed that the signal-plus-noise probability density function has the same standard deviation *i.e.* that the signal itself is not noisy. Under the same assumptions we calculated P_3. The symbol d', introduced by Tanner *et al.* (16) is the ratio of the strength of the signal and the standard deviation of the noise probability density function.

Values of P_3' and P_4 have been determined by experiments using respectively touch stimuli, electrical stimuli, and warmth stimuli applied to the inner side of the forearm. Moreover some preliminary experiments have been carried out on vision using small short light flashes with durations of 20 msec. and a magnitude of 3′ stimulating the dark-adapted eye 10°nasal. The results of these experiments are partially shown in the Figs. 11 and 12. The drawn straight line gives the relation between P_3' and P_4 which holds under the threshold model. The broken line gives the relation under the decision model using the equation $P_3' = 3(P_3 - P_4)$ and the above-mentioned calculated values of P_3 and P_4. It will be seen that the experimental data differ distinctly from the drawn line. The percentage indicated in the figures beside the subject gives the level of significance of the deviation from the straight line. The experiments with the warmth stimuli fit the broken line rather well. The experimental points with touch and electrical stimuli are on the average lower.

The preliminary experiments with light stimuli show also a significant devi-

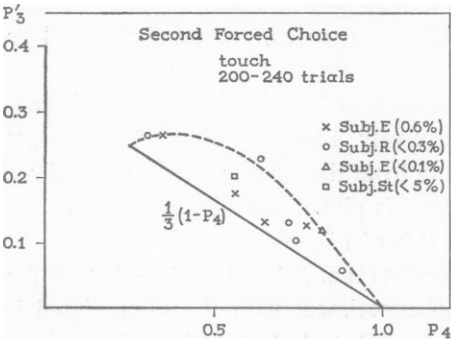

FIGURE 11

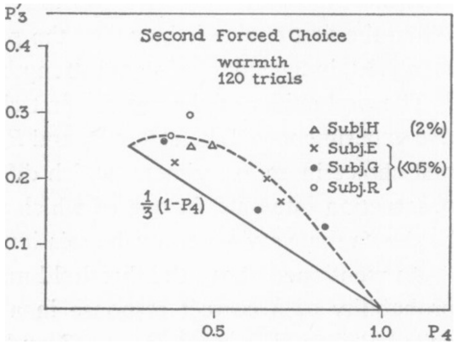

FIGURE 12

FIGURE 11 Probability of correct second forced choice *versus* probability of correct first choice. Four alternatives. Solid line: second choice mere guesses, broken line: theoretical line based on decision model. × and ○, touch; △ and □, electrical stimulation.

FIGURE 12 The same as Fig. 11, but with warmth.

ation of the experimental points from the straight line for one subject, but two other subjects did not.

These experiments show therefore that the threshold model is not adequate. The probability of a correct second choice when the first choice is wrong is significantly higher than the threshold model predicts. This is a strong argument for not considering false positives as mere guesses and accepting the significance of additive noise in the detection of small signals.

DISCUSSION

The especially pronounced occurrence of false positive responses in experiments on the skin senses suggests that internal neural noise exists which is undistinguishable from the neural activity caused by a stimulus. Threshold theory considers these false positives as guesses. It is shown that under the threshold theory the threshold value is dependent on the probability of false positive responses (7), which is inconsistent with the underlying assumptions of this theory.

The second forced choice experiments show clearly that the second choice contains a significant amount of information on the occurrence of a signal. This is a strong argument for accepting this internal noise. Also on the grounds of electrophysiological data it has been suggested that internal noise plays a part in limiting the sensitivity of sense organs. FitzHugh (17, 18) gave a statistical analysis of discharges of single ganglion cells in the retina of the cat and attempted to relate these data with the results of psychophysical measurements of visual sensitivity.

The experimental results with the warmth and touch sense can be explained in first approximation by an internal noise (including the fluctuation of the criterion) which has a gaussian distribution when expressed in terms of input signal. If one assumes that the internal neural activity is also normally distributed then it follows that the neural activity is proportional to signal strength.

Accepting the decision model the measure for the sensitivity of a sensory system is the standard deviation of this gaussian distribution. If the detectability of small signals by a human observer is well described by the decision model this measure is independent of the criterion of the subject and of the psychophysical procedure.

We found a small deviation from the gaussian distribution for very small signals with the touch sense and with electrical stimulation. The experiments with four categories of magnitude of observation show that this deviation is explained by assuming that the internal neural activity is normally distributed and that the relation between signal strength and the thereby caused neural activity is non-linear. This is shown schematically in Fig. 13. If the curve has the course of the dotted line one could speak about a threshold s_t being a value of the stimulus below which no neural activity is generated by the stimulus.

With the touch sense this non-linearity is found using mechanical deformation

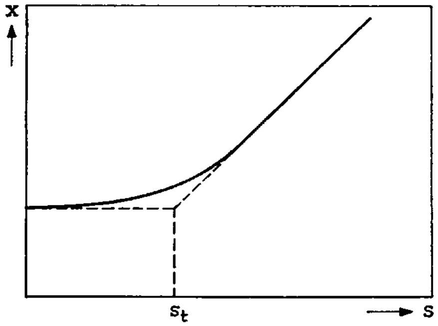

FIGURE 13 Solid line: Non-linear relation between neural activity x and signal strength s. Broken line: Course with a stimulus threshold s_t.

and is even more pronounced with electrical stimulation, but it is not found with the warmth sense. It may be possible that this has something to do with the existence of a constant non-zero rate of action potentials of the warmth fibers when temperature is constant (19), while the touch fibers show a complete adaptation. Change in interval duration between action potentials may be a continuous process, while for the generation of action potentials in the touch fibers the stimulus has to exceed a certain threshold value.

We think it better, however, not to use the word "threshold" for the course of the curve of Fig. 13, but the broader term "non-linearity." First, the possibility of a distinction between the solid and the dotted line in Fig. 13 depends on the accuracy of the experiments and this too on the number of observations. This is obviously very limited in psychophysical experiments. Furthermore the term "threshold" in psychophysical threshold theory has a different meaning because it refers to a cut-off in the continuum of observation.

It can be seen in Fig. 11 that on the average the probability of the correct second forced choice responses for the touch sense and with electrical stimulation is lower than predicted by the decision model. We did not perform further experiments to clarify this point. We can only suggest a possible explanation. One of the observers remarked that in a part of the trials he did not get any sensation and that he thought that the responses on these trials were mere guesses. This would indeed explain the deviation but the decision model does not account for such an additional temporary high cut-off level. Future experiments should elucidate this point further.

An important question is whether besides the internal neural noise of an additive nature a multiplicative noise must also be assumed. This multiplicative noise is a fluctuation which is a monotonic increasing function of signal strength and which is zero when signal strength is zero. Such a noise could be due to fluctuations in the output of the signal generator but also to fluctuations in the transfer function which connects the signal with the neural activity. A multiplicative noise would cause an increase in the variance of the signal-plus-noise density function with signal strength.

As has been mentioned above the experiments on the touch and warmth sense do not justify the assumption of multiplicative noise. But it is very likely that this kind of noise is very pronounced in experiments on the absolute sensitivity of the visual sense organ because the fluctuation in the number of quanta is considerable. The second forced choice experiments have been also carried out in our department on the dark-adapted eye with small light flashes. Only one of the three observers showed a significantly high score of correct second responses. It is very well possible that internal noise in one subject does play a role while in the other this noise is much smaller than the fluctuations in the neural activity due to the statistics of the quanta in the light flash. We have not succeeded as yet in giving a reasonable description of the visual absolute sensitivity measurement.

One can expect that multiplicative noise is important in the measurement of difference limina. Swets *et al.* (5, 7) performed second forced choice experiments on the detectability of light flashes against a background. The results show again that the second response conveys a significant amount of information about the signal. The data points are fitted well by a theoretical curve which is derived on the basis of the decision model assuming that the variance of the signal-plus-noise density function increases proportional with signal strength. This is a kind of multiplicative noise.

Tanner (20) applied the theory of signal detectability to auditory amplitude discrimination. He assumes besides a noise introduced by the experimenter a noise generated by the equipment and a kind of multiplicative noise caused by the amplitude variation in the oscillator being proportional to the power of the lower of two signals to be discriminated. Tanner (20) suggests that the law of Weber should be explained on the basis of this generator inconstancy. This seems, however, not very likely.

In our opinion the most plausible explanation of the law of Weber is given by the assumption that the transfer function between the input signal and the magnitude of sensation fluctuates. The hereby caused fluctuation in the sensation is usually not perceived. The just noticeable change in stimulus has to give rise to a change in sensation which is proportional to the fluctuation in the sensation. A similar assumption is widely accepted in vision. If, furthermore, according to the investigations of Stevens (21), the validity of the law of Plateau is accepted, which states that the sensation magnitude is an exponential function of the stimulus strength, the law of Weber can be easily derived. This derivation and some important consequences will be the object of a separate paper.

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